

de retenție de cca 70 mln m³. de apă la cota nivelului normal 77m și cota la creasta barajului de 84 m. În secțiunea frontului de retenție a Nodul hidrotehnic Nistean 2 (Naslavcea). r. Nistru are o lungime de cca. 20 km, iar partea moldovenească este de 3,9 km din partea dreapta în amonte de baraj pe malul drept. Producerea de energie electrică la hidrocentrala, fiecare cu câte 20,4 MW, cu o energie medie anuală de 100 GWh. Energia produsă este furnizată în mod egal celor două sisteme energetice Md și Ua. Graficul de exploatare a hidrocentralelor este subordonat graficului celorlalte folosințe, furnizând energie în principal în perioada de varf a graficului de sarcină. Debitul minim necesar în aval – este de 100 m³/s și la revarsare în limanul Nistean 80m³/s.

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GREEN TENSOR FOR AN ELASTIC CYCLE CONSTRUCTED BY USING INFLUENCE ELEMENTS METHOD

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Abstract. The Green's tensor for the first (displacement given) boundary value problem of elasticity for a circular domain is computed under a closed form expression and in the form of Fourier series. The method of solution uses the “influence element method” for which the Green's tensor is given by representation using Green's function for Poisson's equation. The main problem to use such a representation is to find the dilatation along the boundary induced by the displacements Green's function. The volume dilatation is then obtained by solving an integral equation along the circular boundary. Explicit expressions are obtained for the Green's displacements tensor and for the traction along the circular boundary, allowing expressing the solution for any kind of “displacement” boundary condition and body forces. On the basis of the constructed Green's tensor is given the integral formula which presents a generalization of the well known Green's integral formula from the theory of harmonic potentials onto the theory of elasticity.

Keywords: Green's functions, Green's tensor, volume dilatation, generalization of Green's integral formula for a cycle, elasticity

INTRODUCTION

In the present paper it is proposed a generalization of classic Green's integral formula for the circle $V(0 \leq r \leq r_0, 0 \leq \varphi \leq 2\pi)$ from the theory of potential harmonics [2, 7]

$$U(r, \varphi) = \int_0^{2\pi} \int_0^{r_0} f(\rho, \psi) G(r, \varphi; \rho, \psi) \rho d\rho d\psi - \int_0^{2\pi} g(\varphi') \frac{\partial G(r_0, \varphi'; r, \varphi)}{\partial n_{r_0}} r_0 d\varphi' \quad (1)$$

onto the first basic problem of the theory of elasticity in form:

$$U_q(r, \varphi) = \int_0^{2\pi} \int_0^{r_0} f_s(\rho, \psi) U_s^{(q)}(r, \rho; \varphi, \psi) \rho d\rho d\psi - \int_0^{2\pi} g_s(\varphi') P_s^{(q)}(r_0, \varphi'; r, \varphi) d\varphi', \quad (2)$$

where $s = r, \varphi$; $q = \rho, \psi$ - are polar coordinates, and the index s is the summing index.

In formulae (1) the functions $G(r, \varphi; \rho, \psi)$ and $\partial G(r_0, \varphi'; r, \varphi) / \partial n_{r_0}$ are Green's functions for Poisson equation [2, 7]:

$$G(r, \varphi; \rho, \psi) = \begin{cases} G_l(r, \varphi; \rho, \psi) = \frac{1}{2\pi} \ln \frac{r_0}{\rho} + \sum_{n=1}^{\infty} \frac{1}{2\pi n} \left[\left(\frac{r}{\rho} \right)^n - \left(\frac{r\rho}{r_0^2} \right)^n \right] \cos n(\varphi - \psi); & r \leq \rho; \\ G_r(r, \varphi; \rho, \psi) = \frac{1}{2\pi} \ln \frac{r_0}{r} + \sum_{n=1}^{\infty} \frac{1}{2\pi n} \left[\left(\frac{\rho}{r} \right)^n - \left(\frac{r\rho}{r_0^2} \right)^n \right] \cos n(\varphi - \psi); & r \geq \rho; \end{cases} \quad (3)$$

$$\frac{\partial G(r_0, \varphi'; r, \varphi)}{\partial n_{r_0}} = -\frac{1}{2\pi r_0} - \frac{1}{\pi r_0} \sum_{n=1}^{\infty} \left(\frac{r}{r_0} \right)^n \cos n(\varphi' - \varphi), \quad (4)$$

- written in the form of Fourier series, and

$$G(r, \varphi; \rho, \psi) = G(M, N) = \frac{1}{2\pi} \ln \sqrt{\frac{r_0^2 - 2r\rho \cos(\varphi - \psi) + (r_0^{-1}r\rho)^2}{r_0^2 - 2r\rho \cos(\varphi - \psi) + \rho^2}}; \quad (5)$$

$$\frac{\partial G(r_0, \varphi'; r, \varphi)}{\partial n_{r_0}} = \frac{1}{2\pi r_0} \cdot \frac{(r^2 - r_0^2)}{r_0^2 - 2rr_0 \cos(\varphi' - \varphi) + r^2}, \quad (6)$$

- written in the closed form.

In the formula (2) $U_q(r, \varphi)$ are the components of displacements in the point $M(r, \varphi)$, generated by the components of volume forces $f_s(\rho, \psi)$ and boundary displacements $g_s(\varphi')$; $U_s^{(q)}(r, \rho; \varphi, \psi)$ are the components of Green's tensor of displacements (the displacements in the inner point $M(r, \varphi)$ in the direction $s = r, \varphi$, generated by the concentrated inner forces $\delta_{sq} \delta(M - N)$ acting in the inner point $N(\rho, \psi)$ in the direction $q = \rho, \psi$). Finally $P_s^{(q)}(r_0, \varphi'; r, \varphi)$ are the components of tractions on the boundary $\Gamma\{r = r_0, 0 \leq \varphi \leq 2\pi\}$ of the circle, generated by the displacements $U_s^{(q)}(r, \rho; \varphi, \psi)$. Finally, $\delta(M - N)$ is the Dirac's function.

MATERIAL AND METHOD

To obtain the integral formula (2) first at all we have constructed the components of displacements $U_s^{(q)}(r, \rho; \varphi, \psi)$ using influence elements method [4, 6]. For this aim we have used the representation of Green's tensor via Green's function for Poisson's equation [5]. The main difficult problem to use such a

representation was to find the dilatation along the boundary induced by the displacements Green's function, which was obtained by solving an integral equation along the circular boundary and in computing some integrals using the books [1, 3]. Then, we have obtained the Green's tensor in the closed form and in the form of Fourier series.

RESULTS AND DISCUSSIONS

Components of Green's Tensor in the Closed Form

The obtained components of Green's tensor of displacements can be written in the following closed form:

$$U_r^{(\rho)}(r, \rho; \varphi, \psi) = \frac{A}{2\pi} \left\{ \left[B \cos(\varphi - \psi) + (r - \rho \cos(\varphi - \psi)) \frac{\partial}{\partial \rho} \right] \ln \frac{\bar{R}}{R} + \frac{B^{-1} r_0^2 \left(1 - \frac{r^2}{r_0^2} \right) \left[\left(B \left(1 + \frac{\rho^2}{r_0^2} \right) + \left(1 - \frac{\rho^2}{r_0^2} \right) \right) \frac{\partial}{\partial r} \ln \bar{R} - \left(1 - \frac{\rho^2}{r_0^2} \right) \frac{\partial^2}{\partial r \partial \varphi} \arctg \bar{F} \right]}{2\rho} \right\}; \quad (7)$$

$$U_r^{(\psi)}(r, \rho; \varphi, \psi) = \frac{A}{2\pi} \left\{ \left[B \sin(\varphi - \psi) + (r - \rho \cos(\varphi - \psi)) \frac{\partial}{\rho \partial \psi} \right] \ln \frac{\bar{R}}{R} - \frac{B^{-1} r_0^2 \left(1 - \frac{r^2}{r_0^2} \right) \left(1 - \frac{\rho^2}{r_0^2} \right) \left[(B+1) \frac{\partial}{\partial r} \arctg \bar{F} + \frac{\partial^2}{\partial r \partial \varphi} \ln \bar{R} \right]}{2\rho} \right\}, \quad (8)$$

for radial displacements, and

$$U_\varphi^{(\rho)}(r, \rho; \varphi, \psi) = \frac{A}{2\pi} \left\{ \sin(\varphi - \psi) \left(\rho \frac{\partial}{\partial \rho} - B \right) \ln \frac{\bar{R}}{R} + \frac{B^{-1} r_0^2 \left(1 - \frac{r^2}{r_0^2} \right) \left[\left(B \left(1 + \frac{\rho^2}{r_0^2} \right) + \left(1 - \frac{\rho^2}{r_0^2} \right) \right) \frac{\partial}{\partial r} \arctg \bar{F} - \left(1 - \frac{\rho^2}{r_0^2} \right) \frac{\partial^2}{\partial r \partial \varphi} \ln \bar{R} \right]}{2\rho} \right\}; \quad (9)$$

$$U_\varphi^{(\psi)}(r, \rho; \varphi, \psi) = \frac{A}{2\pi} \left\{ \left[B \cos(\varphi - \psi) + \sin(\varphi - \psi) \frac{\partial}{\partial \psi} \right] \ln \frac{\bar{R}}{R} - \frac{B^{-1} r_0^2 \left(1 - \frac{r^2}{r_0^2} \right) \left(1 - \frac{\rho^2}{r_0^2} \right) \left[\frac{\partial^2}{\partial r \partial \varphi} \arctg \bar{F} - (B+1) \frac{\partial}{\partial r} \ln \bar{R} \right]}{2\rho} \right\}, \quad (10)$$

for circular displacements.

In expressions (7)-(10) the constants A and B are determined via the Lamé's constants of elasticity λ, μ in the following form:

$$A = (\lambda + \mu) / 2\mu(\lambda + 2\mu); \quad B = (\lambda + \mu) / (\lambda + 3\mu); \quad (11)$$

the functions R, \bar{R}, \bar{F} are determined by the following expressions

$$R = \sqrt{r^2 + 2r\rho \cos(\varphi - \psi) + \rho^2}; \quad \bar{R} = \sqrt{r_0^2 + 2r\rho \cos(\varphi - \psi) + \left(\frac{r\rho}{r_0} \right)^2}; \quad \bar{F} = \frac{r\rho \sin(\varphi - \psi)}{r_0^2 - R\rho \cos(\varphi - \psi)}, \quad (12)$$

and $\delta_{q\rho}, \delta_{q\psi}$ are the Chronicker's

symbols $\delta_{q\rho} = 1, q = \rho; \delta_{q\rho} = 0, q \neq \rho; \delta_{q\psi} = 1, q = \psi; \delta_{q\psi} = 0, q \neq \psi.$

Components of Tractions in the Closed Form

In the formula (2) the components of vector forces on the boundary (tractions), generated by the displacements $U_s^{(q)}(r, \rho; \varphi, \psi)$ are determined in the following closed form:

$$P_r^{(\rho)}(r_0, r; \varphi', \varphi) = -\frac{B^{-1}}{2\pi} \left\{ \frac{Br}{r_0^2} + \left[\left(\frac{r^2}{r_0^2} - 1 \right) - B \left(1 + \frac{r^2}{r_0^2} \right) \right] \frac{\partial}{\partial r} \ln R + \left(\frac{r^2}{r_0^2} - 1 \right) \frac{\partial^2}{\partial \varphi \partial r} \arctg F \right\}; \quad (13)$$

$$P_r^{(\psi)}(r_0, r; \varphi', \varphi) = \frac{B^{-1}}{2\pi} \left(\frac{r^2}{r_0^2} - 1 \right) \frac{\partial}{\partial r} \left[(B+1) \arctg F - \frac{\partial}{\partial \varphi} \ln R \right]; \quad (14)$$

for radial tractions, and

$$P_\varphi^{(\rho)}(r_0, r; \varphi', \varphi) = \frac{A\mu}{2\pi} \left(1 - \frac{r^2}{r_0^2} \right) \left[(B - B^{-1}) \frac{\partial}{\partial r} \arctg F + (1 + B^{-1}) \frac{\partial^2}{\partial r \partial \varphi} \ln R \right]; \quad (15)$$

$$P_\varphi^{(\psi)}(r_0, r; \varphi', \varphi) = \frac{A\mu}{2\pi} \left\{ -(B+1) \frac{r}{r_0^2} + \frac{B}{r_0} \cos(\varphi' - \varphi) + (B+1) \left[2 - B^{-1} + (B^{-1} + 2) \frac{r^2}{r_0^2} \right] \frac{\partial}{\partial r} \ln R + \left(\frac{r^2}{r_0^2} - 1 \right) (B^{-1} + 2) \frac{\partial^2}{\partial r \partial \varphi} \arctg F \right\}; \quad (16)$$

for circular tractions.

In the formulae (13)-(16) the functions R , F are determined as $R = \sqrt{r_0^2 - 2r_0 r \cos(\varphi' - \varphi) + r^2}$ and $F = r \sin(\varphi' - \varphi) / (r_0 - r \cos(\varphi' - \varphi))$.

Components of Green's Tensor in the Form of Fourier Series

The final expressions for the Green's tensor of displacements $U_r^{(\rho)}$, $U_r^{(\psi)}$, $U_\varphi^{(\rho)}$, $U_\varphi^{(\psi)}$ for the cycle will be expressed by the Fourier series in the following form:

$$U_r^{(\rho)} = \tilde{U}_r^{(\rho)} + \frac{AB^{-1}}{4\pi} \left(\frac{r^2}{r_0^2} - 1 \right) \sum_{n=1}^{\infty} \left[B + 1 + (B-1) \frac{\rho^2}{r^2} + n \left(1 - \frac{\rho^2}{r^2} \right) \right] \left(\frac{r\rho}{r_0^2} \right)^{n-1} \cos n(\varphi - \psi); \quad (17)$$

$$U_r^{(\psi)} = \tilde{U}_r^{(\psi)} - \frac{AB^{-1}}{4\pi} \left(\frac{r^2}{r_0^2} - 1 \right) \sum_{n=1}^{\infty} \left[1 - B + (B+1) \frac{\rho^2}{r^2} + n \left(\frac{\rho^2}{r^2} - 1 \right) \right] \left(\frac{r\rho}{r_0^2} \right)^{n-1} \sin n(\varphi - \psi);$$

for radial displacements, and

$$U_\varphi^{(\rho)} = \tilde{U}_\varphi^{(\rho)} + \frac{AB^{-1}}{4\pi} \left(\frac{r^2}{r_0^2} - 1 \right) \sum_{n=1}^{\infty} \left[B + 1 + (B-1) \frac{\rho^2}{r^2} + n \left(1 - \frac{\rho^2}{r^2} \right) \right] \left(\frac{r\rho}{r_0^2} \right)^{n-1} \sin n(\varphi - \psi); \quad (18)$$

$$U_\varphi^{(\psi)} = \tilde{U}_\varphi^{(\psi)} - \frac{AB^{-1}}{4\pi} \left(\frac{r^2}{r_0^2} - 1 \right) \sum_{n=1}^{\infty} \left[1 - B + (B+1) \frac{\rho^2}{r^2} + n \left(\frac{\rho^2}{r^2} - 1 \right) \right] \left(\frac{r\rho}{r_0^2} \right)^{n-1} \cos n(\varphi - \psi).$$

for circular displacements.

In equations (17) and (18) the displacements $\tilde{U}_r^{(\rho)}$, $\tilde{U}_r^{(\psi)}$, $\tilde{U}_\varphi^{(\rho)}$, $\tilde{U}_\varphi^{(\psi)}$ are determined in the form of Fourier series, using the following expressions:

$$\begin{aligned} \tilde{U}_r^{(q)}(r, \varphi; \rho, \psi) = & \frac{A}{2\pi} \left\{ \delta_{q\rho} \left[\tilde{U}_{r0} + \tilde{U}_{r1} \cos(\varphi - \psi) + \sum_{n=1}^{\infty} \tilde{U}_{r,n+1} \cos(n+1)(\varphi - \psi) \right] + \right. \\ & \left. + \delta_{q\psi} \left[\tilde{U}_{r1} \sin(\varphi - \psi) + \sum_{n=1}^{\infty} \tilde{U}_{r,n+1} \sin(n+1)(\varphi - \psi) \right] \right\}; \end{aligned} \quad (19)$$

$$\begin{aligned} \tilde{U}_\varphi^{(q)}(r, \varphi; \rho, \psi) = & \frac{A}{2\pi} \left\{ \delta_{q\rho} \left[\tilde{U}_{\varphi1} \sin(\varphi - \psi) + \sum_{n=1}^{\infty} \tilde{U}_{\varphi,n+1} \sin(n+1)(\varphi - \psi) \right] + \right. \\ & \left. + \delta_{q\psi} \left[\tilde{U}_{\varphi0} + \tilde{U}_{\varphi1} \cos(\varphi - \psi) + \sum_{n=1}^{\infty} \tilde{U}_{\varphi,n+1} \cos(n+1)(\varphi - \psi) \right] \right\}. \end{aligned} \quad (20)$$

- for the displacements $\tilde{U}_r^{(q)}$, $\tilde{U}_\varphi^{(q)}$.

In the series (19) and (20) the coefficients of expansion are the functions of polar coordinates ρ, ψ and they are determined by the following expressions:

$$\tilde{U}_{r0} = \frac{B-1}{2} \begin{cases} (r/\rho) - (r\rho/r_0^2), & r \leq \rho; \\ (\rho/r) - (r\rho/r_0^2), & r \geq \rho. \end{cases} \quad (21)$$

$$\tilde{U}_{r1} = \begin{cases} B \ln\left(\frac{r_0}{\rho}\right) + 1 + \frac{B-2}{4} \left[\left(\frac{r}{\rho}\right)^2 - \left(\frac{r\rho}{r_0^2}\right)^2 \right] - \left(\frac{r}{r_0}\right)^2; & r \leq \rho; \\ B \ln\left(\frac{r_0}{r}\right) + 1 + \frac{B-2}{4} \left[\left(\frac{\rho}{r}\right)^2 - \left(\frac{r\rho}{r_0^2}\right)^2 \right] - \left(\frac{r}{r_0}\right)^2; & r \geq \rho. \end{cases} \quad (22)$$

$$\tilde{U}_{r,n+1} = \begin{cases} \frac{B+n}{2n} \left(\frac{r}{\rho}\right)^n - \frac{B-n}{2n} \left(\frac{r\rho}{r_0^2}\right)^n + \frac{B-(n+2)}{2(n+2)} \left[\left(\frac{r}{\rho}\right)^{n+2} - \left(\frac{r\rho}{r_0^2}\right)^{n+2} \right] - \left(\frac{r}{\rho}\right)^2 \left(\frac{r\rho}{r_0^2}\right)^n; & r \leq \rho; \\ \frac{B+n}{2n} \left(\frac{\rho}{r}\right)^n - \frac{B-n}{2n} \left(\frac{r\rho}{r_0^2}\right)^n + \frac{B-(n+2)}{2(n+2)} \left[\left(\frac{\rho}{r}\right)^{n+2} - \left(\frac{r\rho}{r_0^2}\right)^{n+2} \right] - \left(\frac{r}{\rho}\right)^2 \left(\frac{r\rho}{r_0^2}\right)^n; & r \geq \rho \end{cases} \quad (23)$$

- for the coefficients of series for displacements $\tilde{U}_r^{(\rho)}$ in eqn. (19);

$$\tilde{U}_{r1} = \begin{cases} B \ln\left(\frac{r_0}{\rho}\right) - \frac{B-2}{4} \left(\frac{r}{\rho}\right)^2 + \frac{B+2}{4} \left(\frac{r\rho}{r_0^2}\right)^2 - \left(\frac{r}{r_0}\right)^2; & r \leq \rho; \\ B \ln\left(\frac{r_0}{r}\right) - \frac{B+2}{4} \left[\left(\frac{\rho}{r}\right)^2 - \left(\frac{r\rho}{r_0^2}\right)^2 \right] - \left(\frac{r}{r_0}\right)^2; & r \geq \rho. \end{cases} \quad (24)$$

$$\tilde{U}_{r,n+1} = \begin{cases} \frac{B-n}{2n} \left[\left(\frac{r}{\rho}\right)^n - \left(\frac{r\rho}{r_0^2}\right)^n \right] - \frac{B+n+2}{2(n+2)} \left[\left(\frac{r}{\rho}\right)^{n+2} - \left(\frac{r\rho}{r_0^2}\right)^{n+2} \right] + \left(\frac{r}{\rho}\right)^{n+2} - \frac{r}{\rho} \left(\frac{r\rho}{r_0^2}\right)^{n+1}; & r \leq \rho; \\ \frac{B-n}{2n} \left[\left(\frac{\rho}{r}\right)^n - \left(\frac{r\rho}{r_0^2}\right)^n \right] + \frac{B+n+2}{2(n+2)} \left[\left(\frac{\rho}{r}\right)^{n+2} - \left(\frac{r\rho}{r_0^2}\right)^{n+2} \right] + \left(\frac{\rho}{r}\right)^n - \frac{r}{\rho} \left(\frac{r\rho}{r_0^2}\right)^{n+1}; & r \geq \rho \end{cases} \quad (25)$$

- for the coefficients of series for displacements $\tilde{U}_r^{(\psi)}$ in eqn. (19);

$$\bar{U}_{\varphi 1} = \left\{ \begin{array}{l} -1 - B \ln \frac{r_0}{\rho} + 1 + \frac{B+2}{4} \left(\frac{r}{\rho} \right)^2 - \frac{B-2}{4} \left(\frac{r\rho}{r_0^2} \right)^2; r \leq \rho; \\ -B \ln \frac{r_0}{r} + 1 + \frac{B-2}{4} \left[\left(\frac{\rho}{r} \right)^2 - \left(\frac{r\rho}{r_0^2} \right)^2 \right]; r \geq \rho. \end{array} \right\}; \quad (26)$$

$$\bar{U}_{\varphi, n+1} = \left\{ \begin{array}{l} -\frac{B+n}{2n} \left(\frac{r}{\rho} \right)^n + \frac{B-n}{2n} \left(\frac{r\rho}{r_0^2} \right)^n + \frac{B+n+2}{2(n+2)} \left(\frac{r}{\rho} \right)^{n+2} - \frac{B-(n+2)}{2(n+2)} \left(\frac{r\rho}{r_0^2} \right)^{n+2}; r \leq \rho; \\ -\frac{B-n}{2n} \left[\left(\frac{\rho}{r} \right)^n - \left(\frac{r\rho}{r_0^2} \right)^n \right] + \frac{B-(n+2)}{2(n+2)} \left[\left(\frac{\rho}{r} \right)^{n+2} - \left(\frac{r\rho}{r_0^2} \right)^{n+2} \right]; r \geq \rho \end{array} \right\}; \quad (27)$$

- for the coefficients of series for displacements $\tilde{U}_{\varphi}^{(\rho)}$ in eqn. (20), and finally

$$\tilde{U}_{\varphi 0} = (B+1) \left\{ \begin{array}{l} \left(\frac{r}{\rho} \right) - \left(\frac{r\rho}{r_0^2} \right); r \leq \rho; \\ \left(\frac{\rho}{r} \right) - \left(\frac{r\rho}{r_0^2} \right); r \geq \rho. \end{array} \right\}; \quad (28)$$

$$\tilde{U}_{\varphi 1} = \left\{ \begin{array}{l} B \ln \left(\frac{r_0}{\rho} \right) + \frac{B+2}{2} \left[\left(\frac{r}{\rho} \right)^2 - \left(\frac{r\rho}{r_0^2} \right)^2 \right]; r \leq \rho; \\ B \ln \left(\frac{r_0}{r} \right) + \frac{B+2}{2} \left[\left(\frac{\rho}{r} \right)^2 - \left(\frac{r\rho}{r_0^2} \right)^2 \right] - \left(\frac{r}{r_0} \right)^2; r \geq \rho. \end{array} \right\}; \quad (29)$$

$$\tilde{U}_{\varphi, n+1} = \left\{ \begin{array}{l} \frac{B-n}{n} \left[\left(\frac{\rho}{r} \right)^n - \left(\frac{r\rho}{r_0^2} \right)^n \right] + \frac{B+n+2}{(n+2)} \left[\left(\frac{r}{\rho} \right)^{n+2} - \left(\frac{r\rho}{r_0^2} \right)^{n+2} \right]; r \leq \rho; \\ \frac{B-n}{n} \left[\left(\frac{r}{\rho} \right)^n - \left(\frac{r\rho}{r_0^2} \right)^n \right] + \frac{B+n+2}{(n+2)} \left[\left(\frac{\rho}{r} \right)^{n+2} - \left(\frac{r\rho}{r_0^2} \right)^{n+2} \right]; r \geq \rho \end{array} \right\}; \quad (30)$$

for the coefficients of series for displacements $U_{\varphi}^{(\psi)}$ in eqn. (20).

At the end of this section we must notice that obtained both in closed form and in the form of Fourier series the Green's tensor satisfy all equations for the first basic boundary value problem of theory of elasticity and also the Maxwell's reciprocity theorem.

CONCLUSIONS

The formula (2) represents the generalization of the Green's integral formula for the cycle from theory of harmonic potentials (1) to theory of elasticity.

The advantage of proposed formula (2) is that it represent the searched displacements directly via integrals of given inner body forces, of on the boundary circumference displacements and known already kernels: (7)-(12) - for displacements written in closed form; (17)-(30) - for displacements, written in the form of Fourier series, and (13)-(16) - for the tractions, written in the closed form. Also, this formula permits us to solve many boundary value problems of elasticity in integral form for different laws of given inner body forces and of given boundary displacements.

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MODERNIZAREA CU SCOPUL REDUCERII CONSUMULUI DE ENERGIEI ELECTRICE A STAȚIILOR DE POMPARE LA ÎS „ACVA- NORD”

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Abstract: This method of obtaining electric energz allws reduces expenses for capital structures and operation. In case hzdropower units of emerge-cz they can supply the pumping stations so that the technological and operational flux need not to be interrupted

Key words: Energy, Pumps, Station, Test, Water.

INTRODUCERE

Acum pomparea apei de la priză r. Nistru pînă la STA și de la rezervoarele cu apa potabilă spre rezervorul de sus se asigură cu numai cu un agregat mare (D4000-95) la fiecare din 4 stații de pompare. Din informația primită de la Beneficiar și observări petrecute la fața locului agregate se pornesc 3-4 ori în 24h cu o durată a lucrului de la 1,5-2 h. Procesul de pornire din cercetările autorului și informația primită de la colaboratori de la AdSB (ÎIS”AN”) și AC Bălți: operatorul de la Rezervoarele $2 \times 6000 \text{m}^3 = 12000 \text{m}^3$ la cota terenului 170m de la intrare în or. Bălți cînd observă ca adîncimea în rezervoare atinge valoarea 2,8m prin telefon informează dispeceratele AC Bălți și Ad Soroca-Bălți (ÎIS „AcvaNord” amplasată în or.Soroca). Ca urmare treptat se pun în funcțiune toate 4 Sp începînd de la Sp1 pînă la Sp4. Apa nimereste în rezervoarele de sus $2 \times 2000 = 4000 \text{m}^3$ la cota terenului 303m (în apropierea s.Vanțena). Din care prin conducta de $d1200 \text{mm}$ ($L=18,3 \text{km}$) și cu $d1000 \text{mm}$ ($L=25,9 \text{km}$) transportă apa la rezervoarele de sus 12000m^3 la cota 170m. De aici sub gravitație apa se distribuiește la consumatori din or. Bălți. Aici trebuie de subliniat că una din două conducte tot pri gravitație este transportată spre SpCopaceanca (cota 105m, co rezervor